Last week we learned how to handle inequalities with many factors i.e. inequalities of the form $(x – a)(x – b)(x – c)(x – d) > 0$. This week, let's see what happens in cases where the inequality is not of this form but can be manipulated and converted to this form. We will look at how to handle various complications.

Complication No. 1: $(a – x)(x – b)(x – c)(x – d) > 0$

We want our inequality to be of the form $(x – a)$, not $(a – x)$ because according to the logic we discussed last week, when x is greater than a, we want this factor to be positive. The manipulation involved is pretty simple: $(a – x) = -(x – a)$

So we get: $– (x – a)(x – b)(x – c)(x – d) > 0$

But how do we handle the negative sign in the beginning of the expression? We want the values of x for which the negative of this expression should be positive. Therefore, we basically want the value of x for which this expression itself (without the negative sign in the beginning) is negative.

We can manipulate the inequality to $(x – a)(x – b)(x – c)(x – d) < 0$

Or simply, multiply $– (x – a)(x – b)(x – c)(x – d) > 0$ by -1 on both sides. The inequality sign flips and you get $(x – a)(x – b)(x – c)(x – d) < 0$

e.g. Given: $(4 – x)(2 – x)(-9 – x) < 0$

We can re-write this as $–(x – 4)(2 – x)(-9 – x) < 0$

$(x – 4)(x – 2)(-9 – x) < 0$

$-(x – 4)(x – 2)(x + 9) < 0$

$(x – 4)(x – 2)(x – (-9)) > 0$ (multiplying both sides by -1)

Now the inequality is in the desired form.

Complication No 2: $(mx – a)(x – b)(x – c)(x – d) > 0$ (where m is a positive constant)

How do we bring $(mx – a)$ to the form $(x – k)$? By taking m common!

$(mx – a) = m(x – a/m)$

The constant does not affect the sign of the expression so we don't have to worry about it.

e.g. Given: $(2x – 3)(x – 4) < 0$

We can re-write this as $2(x – 3/2)(x – 4) < 0$

When considering the values of x for which the expression is negative, 2 has no role to play since it is just a positive constant.

Now let's look at a question involving both these complications.

Question 1: Find the range of x for which the given inequality holds.

$-2x^3 + 17x^2 – 30x > 0$

Solution:

Given: $-2x^3 + 17x^2 – 30x > 0$

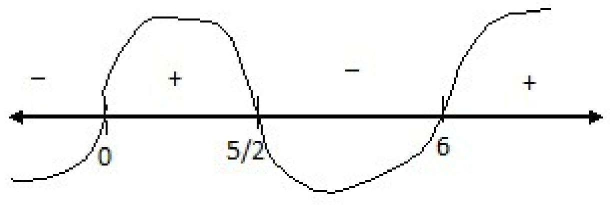
$x(-2x^2 + 17x – 30) > 0$ (taking x common)

$x(2x – 5)(6 – x) > 0$ (factoring the quadratic)

$2x(x – 5/2)(-1)(x – 6) > 0$ (take 2 common)

$2(x – 0)(x – 5/2)(x – 6) < 0$ (multiply both sides by -1)

This inequality is in the required form. Let's draw it on the number line.



We are looking for negative value of the expression. Look at the ranges where we have the negative sign.

The ranges where the expression gives us negative values are $5/2 < x < 6$ and $x < 0$.

Hence, the inequality is satisfied if x lies in the range $5/2 < x < 6$ or in the range $x < 0$.

Plug in some values lying in these ranges to confirm.

Next week, we will look at some more variations which can be brought into this form.